INTRODUCTION
Applied vocational disciplines use many equations that often have several variables. These equations usually give the value of one variable in terms of other variables. When such equations have meanings we can describe in words, we call them literal equations.

SOLVING RATIONAL EQUATIONS
The first type of literal equations we consider here are called rational equations. Rational equations involve the rational mathematical operations only: addition, subtraction, multiplication and division.

Suppose an object moves $D$ meters in a straight line for $t$ seconds. If its initial velocity is $v_0$ m/s and it accelerates at a constant rate, then its final velocity can be found by solving the equation below for $v$.

$$D = \frac{t}{2}(v_0 + v)$$

Let’s walk through a process that can be used to solve many similar rational equations.

I. Simplify each side of the equation - use the distributive property and combine like terms.

In the given equation, there is a pair of parentheses with a factor in front that can be distributed.

$$D = \frac{t}{2}v_0 + \frac{t}{2}v$$

II. Clear fractions by multiplying each side by the LCM of each denominator.

Put more simply, we will multiply each side of the equation by a value that will cancel each denominator. For the equation above, multiplying by 2 will cancel the 2s in the denominators.

$$2D = 2\left(\frac{t}{2}v_0 + \frac{t}{2}v\right)$$

Of course, the process often requires distributing, then reducing fractions.

$$2D = 2 \cdot \frac{t}{2}v_0 + 2 \cdot \frac{t}{2}v$$

So the result looks like this:

$$2D = tv_0 + tv$$

III. Add or subtract to move terms with the desired variable to one side. Other terms go to the other side.

Remember, we want to solve for $v$. The second term on the right side contains this variable.
To isolate it, we subtract the other term from each side.

\[
2D = tv_0 + tv \\
-tv_0 = -tv_0 \\
\hline
2D - tv_0 = tv
\]

One step that could be performed at this point (but it is not required) is to apply the symmetric property of equations: if \( a = b \), then \( b = a \). This results in moving the desired variable to the left.

\[
tv = 2D - tv_0
\]

**IV. Combine like terms.**

There are no terms that can be combined in this equation, so we skip this step.

**V. Factor the desired variable away from all terms containing it.**

There is only one term with \( v \), so it does not need to be factored away.

**VI. Use Equation Multiplication or Division to isolate the desired variable.**

Our variable, \( v \), is multiplied by \( t \), so we divide by this factor. The distributive property tells us that all terms get divided by this on each side of the equation. Remember – if an entire numerator or denominator is cancelled, it is replaced by a 1.

\[
\frac{tv}{t} = \frac{2D}{t} - \frac{tv_0}{t} \\
\]

With some cleanup, we arrive at the equation below.

\[
v = \frac{2D}{t} - v_0
\]

The resulting equation is equivalent to the original, but it gives the final velocity after constant acceleration. For example, if we accelerate from 50m/s, travelling 500m in 4sec, the final velocity is

\[
v = \frac{2D}{t} - v_0 \\
= \frac{2 \cdot 500}{4} - 50 \\
= 200 \text{ m/sec}
\]

The object is moving at 200 meters per second at the end of 4 seconds.
SECTION 1.5
SOLVING LITERAL EQUATIONS

YOUR TURN!
A trapezoid is a four-sided polygon with a pair of opposite sides (the bases) that are parallel. The area of a trapezoid is given by the formula,

\[ A = \frac{h}{2}(B + b). \]

The variables have the following meanings: \( A \) = area, \( B \) & \( b \): larger & smaller of the parallel bases, \( h \) = height.

1. Solve the area formula for the small base, \( b \). Some steps in the process above are not necessary.
   a. Simplify each side of the equation - use the distributive property and combine like terms.
   
   b. Clear fractions by multiplying each side by the LCM of each denominator.
   
   c. Add or subtract to move terms with the desired variable to one side. Other terms go to other side.
   
   d. Combine like terms (if necessary).
   
   e. Factor the desired variable away from all terms containing it (if necessary).
   
   f. Use Equation Multiplication or Division to isolate the desired variable.
2. Suppose a trapezoid has area equal to 33 in$^2$, is 6 in high, and one base is 4 in long. What is the length of the other base?

Many literal equations are simply proportionalities – where quantities are related through multiplication by rates. One example of this is the ideal gas law from chemistry. One form of this law can be expressed as:

$$\frac{P}{T} = \frac{nR}{V}$$

In this equation, $P$ is the pressure of a quantity of gas molecules, $T$ is the temperature, $V$ is the volume, $n$ and $R$ are constants of proportionality.

Suppose we want to solve for the volume, $V$. Most of the steps for solving a rational equation are unnecessary for such problems. If we scan through the steps, the first is not necessary (distribute/simplify), but the second step (clear fractions) seems like a good plan.

We need to clear $T$ and $V$ in the denominators, so we multiply both sides by each.

$$TV \frac{P}{T} = TV \frac{nR}{V}$$

A little cleanup gives a more commonly encountered version of the equation.

$$PV = nRT$$

The third, fourth, and fifth steps are unnecessary. For the sixth step, we want to isolate the volume, $V$, so we divide by the pressure (which is present through multiplication).

$$\frac{PV}{P} = \frac{nRT}{P}$$

The solution is $V = \frac{nRT}{P}$. 

SECTION 1.5
SOLVING LITERAL EQUATIONS

YOUR TURN!
The higher an object is above the earth, the more potential energy it has. If an astronaut of mass $m$ is floating above a planet with mass $M$ at a distance of $r$, the astronaut’s potential energy is given by

$$P = -\frac{GMm}{r},$$

where $G$ is a constant of proportionality.

3. Solve this equation for the mass of the planet, $M$.

To wrap up our discussion, here is a quick overview of the usual process for solving rational equations. Remember, the primary rule for solving equations is that whatever operations are applied must be applied to both sides. What we refer to as the usual process is just a suggested process.

CHECK IT OUT!
The usual process for solving rational equations is as follows.

I. Simplify each side of the equation - use the distributive property and combine like terms.
II. Clear fractions by multiplying each side by the LCM of each denominator.
III. Add or subtract to move terms with the desired variable to one side. Other terms go to the other side.
IV. Combine like terms.
V. Factor the desired variable away from all terms containing it.
VI. Use Equation Multiplication or Division to isolate the desired variable.
QUADRATIC EQUATIONS

The next type of equation we will study is called quadratic. Quadratic equations have a squared variable. To solve a quadratic equation, we reverse the squaring process with a square root. The main thing to remember is that every positive number is associated with two numbers (one positive and the other negative) whose squares are equal to the positive number.

4. Solve the equation, \( x^2 = 9 \). Note that there are two solutions.

What you have discovered here is always an issue when solving an equation whose variable is squared – there can be more than one solution. The property that helps us deal with this is the square root property.

CHECK IT OUT!
The square root property states that if

\[ u^2 = a \]

then

\[ u = \pm \sqrt{a}. \]

The simplest of quadratic equations require nothing more than the following steps.

- Isolate the squared variable with the usual process.
- Apply the square root property.
- The result is two equations that can be split apart and solved individually.

For example, the force between two electrically charged particles is given by

\[ F = \frac{kQq}{r^2}. \]

The charges on the particles are \( Q \) and \( q \), \( r \) is the distance between them, \( F \) is the force, and \( k \) is a constant of proportionality. We want to solve for \( r^2 \). The first step of the usual process that we can actually apply (there are no parentheses) is to clear fractions. To do this, we multiply by the denominator, \( r^2 \).

\[ r^2 \cdot F = r^2 \cdot \frac{kQq}{r^2} \]

Simplifying gives the following equation.

\[ r^2 F = kQq \]
Remember, we want to isolate $r^2$. It is multiplied by $F$, so we need to divide by $F$.

$$\frac{r^2 \cdot \mathcal{E}}{\mathcal{E}} = \frac{kQq}{F}$$

So, we have isolated $r^2$.

$$r^2 = \frac{kQq}{F}$$

All we need to do now is apply the square root property: if $u^2 = a$, then $u = \pm \sqrt{a}$.

$$r = \pm \sqrt{\frac{kQq}{F}}$$

**YOUR TURN!**

The kinetic energy, $K$, of an object with mass $m$ and velocity $v$ is given by

$$K = \frac{1}{2}mv^2.$$  

5. Solve for the velocity, $v$.

---

**Radical Equations**

Solving an equation with a square root is similar to solving an equation with a square. The process is straightforward.

- Isolate the square root with the usual method.
- Square both sides and solve.

6. For example, the equation $T = 2\pi \sqrt{\frac{m}{k}}$ gives the period, $T$, of vibration for an object of mass, $m$, attached to a spring whose spring constant is $k$. Solve the equation for the mass, $m$. 

©2013 SCOTT GUTH
PRACTICAL INTERMEDIATE ALGEBRA - IN CLASS LESSONS
SECTION 1.5
SOLVING LITERAL EQUATIONS

PROPORTIONS AS EQUATIONS

We have solved proportionalities through multiplication by rates. Direct proportions are always expressible in the form

\[ y = mx. \]

The variable, \( x \), is a value that is converted to \( y \) through multiplication by the rate, \( m \). Sometimes, after determining the rate, we like to know the equation for its own sake, where \( x \) and \( y \) remain as variables. Knowing the equation allows us to describe the algebraic relationship between the variables.

For example, suppose you buy 12.3 gallons of gas for $50.92. We know that these quantities are proportional to each other, so we can define an equation that relates them.

If \( x \) = amount of gas, and \( y \) = cost, we can substitute into the equation, \( y = mx \), to solve for the rate, \( m \).

\[
\begin{align*}
y &= mx \\
50.92 &= m \cdot 12.3 \\
\frac{50.92}{12.3} &= \frac{m \cdot 12.3}{12.3} \\
4.14 &= m
\end{align*}
\]

The gas price is $4.14 per gallon. But this is not really the point – what we really want is an equation that relates the variables, \( x \) and \( y \). Since \( m = 4.14 \), and the equation is \( y = mx \), the relationship is

\[ y = 4.14x. \]

This relationship is about something bigger than just my one purchase. It gives information about any purchase. For instance, if I want to know the cost of 17 gallons of gas, I just evaluate the equation using \( x = 17 \).

\[
\begin{align*}
y &= 4.14 \cdot 17 \\
&= 70.38
\end{align*}
\]

The cost for 17 gallons of gas will be $70.38.
SECTION 1.5

SOLVING LITERAL EQUATIONS

YOUR TURN!

7. When driving at a constant speed, the distance driven is directly proportional to time. Suppose that you drive for 27.5 hours at a constant speed, covering a distance of 1430 miles.
   
   a. If $x = \text{time}$ and $y = \text{distance}$, give an equation that represents the relationship between the variables. This includes finding the constant of proportionality, $m$.

   
   b. Using the equation above, what distance would be traveled in 13.9 hours at this speed?

INVERSE PROPORTIONS

When two variables, $x$ and $y$, are inversely proportional, they are related by an equation of the form,

$$y = \frac{k}{x}$$

where $k$ is the constant of proportionality.

8. For example, when a syringe of air is compressed, the volume, $x$, of air is inversely proportional to the air pressure, $y$. When there are 10mL of air, the air pressure is 2.5 atm (atmospheres).

   a. Give an equation that represents the relationship between $x$ and $y$. This includes finding the constant of proportionality, $k$.

   
   b. Use the equation to find the pressure if the air is compressed to 3mL.