The file contains a text about solving linear systems in the context of a pharmacist mixing solutions to prepare an intravenous (I.V.) solution for a patient. The patient's doctor ordered a 250mL solution containing 400mg of dopamine, but the pharmacist only has 200mg per 250mL and 500mg per 250mL solutions in stock. She plans to mix the solutions together in the correct amounts to make a 250mL solution with the right concentration. The text calculates the amounts of each solution needed to make a 250mL solution with 400mg of dopamine by setting up and solving a system of linear equations.
CHECK IT OUT!

A system of two linear equations with two unknowns can always be expressed as

\[
\begin{align*}
ax + by &= c \\
dx + ey &= f
\end{align*}
\]

A solution of such a system is a pair of values, \( (x, y) \) that satisfies both equations.

Before we learn the correct amount of each solution for the mixture, we need to understand how to solve systems of equations.

YOUR TURN!

1. Is the pair \( (3, -1) \) a solution to the system below? Check both equations.

\[
\begin{align*}
2x + y &= 5 \\
4x - 2y &= 14
\end{align*}
\]

One way to solve a linear system of two equations in two unknowns is to graph the equations. Each graph represents all solution of its equation. If the graphs cross, then the intersection point, \( (x, y) \), is a solution to both equations.

2. Solve the first equation in question 1 for \( y \) and graph it below.
3. Solve the second equation in question 1 for \( y \) and graph it on the axes in question 2.

4. You have graphed two lines. Give the point where they intersect. Does this match the solution verified in question 1?

5. Think about how two lines can be oriented in a plane. Is it possible for a system of two linear equations with two unknowns to have no solution? Describe or sketch such a situation below.

6. Is it possible for a system of two linear equations with two unknowns to have more than one solution? Think carefully and describe or sketch such a situation below.

7. Find the solution of the system below by graphing. What point satisfies both equations?

\[
\begin{align*}
2x - y &= 2 \\
4x - 2y &= 6
\end{align*}
\]
8. Find the solution of the system below by graphing. What point satisfies both equations?

\[
\begin{align*}
2x - y &= 2 \\
4x - 2y &= 4 
\end{align*}
\]

9. Describe how a system of 2 linear equations with 2 unknowns has the following.

a. One solution point:

b. No solution points:

c. Infinitely many solution points:

MOVING ON!

We need more efficient ways of solving systems of equations. A method that is rather straightforward is the method of substitution.

CHECK IT OUT!

The method of substitution includes steps that can be summarized as follows.

I. Solve one equation for a variable.

II. Substitute this result into the other equation.

III. This resulting equation has only one variable. Solve it.

IV. Substitute the last result into the equation from step I and solve for the remaining variable.

V. Check your answer by substituting both values into the original system.
YOUR TURN!

10. Let’s solve the following system.

\[
\begin{align*}
2x + y &= 7 \\
3x - 4y &= -6
\end{align*}
\]

I. Solve one equation for a variable. Pick the easiest situation.

II. Substitute this result into the other equation.

III. This resulting equation has only one variable. Solve it.

IV. Substitute the last result into the equation from step I and solve for the remaining variable.

V. Check your answer by substituting both values into the original system.
Moving On!

The next method for solving linear systems of equations is called the method of elimination. The method of elimination can be easier than the substitution method, but it's easiest to describe by example.

\[
\begin{align*}
3x + 2y &= 4 \\
2x + 3y &= 11
\end{align*}
\]

The method of substitution leads to fractions. You will see that the method of elimination avoids them.

When we use the elimination method, we multiply equations by values to make the chosen variable’s coefficients opposite to one another. The equations are then added, and the chosen variable is eliminated.

11. Suppose we want to eliminate \( x \) from the system of equations. What is the smallest value into which both \( x \)-coefficients will divide?

12. Multiply the first equation by 2, then multiply the second equation by -3 below.

\[
\begin{align*}
2 \cdot \text{Eq}_1: \\
-3 \cdot \text{Eq}_2:
\end{align*}
\]

13. Add the equations above by combining like terms below the line.

14. If all went well, you should have a new equation without \( x \). It is eliminated. Solve this equation for \( y \).

15. Take the value for \( y \), and substitute into either of the original equations, and solve for \( x \).
16. Give the solution as an ordered pair.

\[(x, y) = \underline{\underline{\ldots}}\]

17. Check your solution by substituting into each of the original equations.

18. Solve the following system by elimination.

\[
\begin{align*}
4x - 5y &= 1 \\
2x + 3y &= 17
\end{align*}
\]

19. Solve the system of equations for the pharmacist’s mixture at the beginning of this lesson.
MOVING ON!

Sometimes we need to solve linear systems with more than 2 unknowns. To solve one, we need the same number of independent equations as unknowns, and the equations must not be contradictory. In such cases, the method of elimination works well. Consider the system below.

\[
\begin{align*}
2x + y - z &= 0 \\
3x - y - 2z &= 1 \\
x + 2y + 2z &= 6
\end{align*}
\]

We need to pick a variable to eliminate by combining two different pairs of equations. Suppose you wish to eliminate \( y \).

20. Work with your group to combine two equations in a way that eliminates \( y \). Make sure you all agree.

21. Again, combine a different pair of equations in a way that eliminates \( y \). Work with your group.

22. You have two equations in \( x \) and \( z \). This is a system of 2 equations with two unknowns. Solve it with elimination or substitution. Substitute the resulting values into one of the original equations to find \( y \).

23. Check your solution by substituting into each of the three original equations.