SECTION 8.1
REVIEW OF PROPORTIONS AND LITERAL EQUATIONS

INTRODUCTION
This lesson is a review of the more critical methods discussed in our course. These include solving proportionalities and equations.

Let’s begin by practicing a few problems with proportionalities.

YOUR TURN!
1. If I spent $85 filling a 22 gallon tank, how much gas can I buy for $29?
2. Suppose that €15 has the value of $14.10 in the U.S. If gas costs $4.25 per gallon, what is the equivalent rate in Euros per liter (1 gallon is equal to about 3.79 liters).

MOVING ON!
Next, let’s think about solving equations. We have used the usual process for solving rational equations many times. The process usually includes (1) distributing to remove parentheses, (2) clearing fractions, (3) combining like terms, (4) isolating terms containing the desired variable on one side and combining these, and (5) dividing to solve. Use this process to solve the following equations.

3. \[
\frac{2}{5} w - 2 \left( \frac{3}{10} - \frac{1}{5} w \right) = \frac{w}{10} - \frac{3}{5}
\]
4. \[ \frac{5}{14s} + \frac{1}{21} = \frac{9}{7s} \]

Many literal equations use the usual process, but are simpler in form. These often use only multiplication and division to solve.

5. The capacitance of a parallel plate capacitor is given by \( C = \kappa \varepsilon \frac{A}{d} \). Solve for the distance between the plates, \( d \).

6. When two capacitors are connected in-line, the combined capacitance is determined by adding the reciprocals of the capacitances for each: \[ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \]. Solve for \( C \), the combined capacitance.

**MOVING ON!**

We have discussed two types forms of quadratic equations. The first has only one instance of the variable. Such equations can be expressed as \( u^2 = a \) by isolating the squared variable. The equation is solved by applying the square root property, \( u = \pm \sqrt{a} \).

7. Solve the quadratic equation: \( 2(x + 4)^2 - 5 = 27 \).
8. Electric power is computed using the formula, \( P = I^2 R \). Solve for the electric current, \( I \).

Another type of quadratic equation has two instances of the variable and can always be expressed in standard form: \( ax^2 + bx + c = 0 \). We can solve by completing the square if we move the constant to the other side, divide by the \( x^2 \) coefficient, and adding \((\frac{1}{2} \cdot x\text{-coefficient})^2\) to each side. The resulting quadratic can be factored and solved by the square root property.

9. Solve by completing the square: \( 2x^2 - 8x - 10 = 0 \).

We also solve quadratic equations \((ax^2 + bx + c = 0)\) with the quadratic formula,

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

10. Solve the equation from question 9 with the quadratic formula.
Moving On!
Radical equations are simple to solve. When an equation has a variable in a square root, we use the usual process to isolate the radical, then square both sides and solve.

11. Solve the equation $3\sqrt{x + 2} - 5 = 4$.

12. A margin of error formula in statistics is $E = T \frac{s}{\sqrt{n}}$. Solve for the sample size, $n$.

Moving On!
Exponential equations have a variable in an exponent. These are solved by isolating the exponential expression and applying a natural logarithm (ln) to each side. When the exponential is not base $b$, we apply the property that $\ln(b^u) = u\ln(b)$.

13. The compound interest formula is $A = P \left(1 + \frac{r}{n}\right)^{nt}$. If you deposit $3000 at a fixed 8% interest rate, compounded quarterly, when will the account contain $5000?
When an equation has an exponential that is base $e$, we use the same process, but the result is simpler since \( \ln(e^u) = u \). The equation for continuous exponential growth is \( A = A_0e^{kt} \). The instantaneous growth/decay ratio \( (k) \) is positive for growth, and negative for decay.

14. Suppose a population grows exponentially at a 10% annual rate. If it starts at a size of 1200, when will it reach 3000?

Finally, we sometimes encounter equations with logarithms (especially when working with values measured with a logarithmic scale). We use the usual process to isolate the logarithm, so that the equation looks like \( \log_b(u) = a \). This can be solved by asking the question “$b$ to the what is $u$?” The answer \( (a) \) should be written as an equation \( (b^a = u) \), and solved.

15. Solve the equation: \( 5 \log_2(2x) - 2 = 23 \).