Math 110 Chapter 7 Review Problems

1. The president's voter approval is 45%. If you pick 500 voters randomly, what is the probability that less than 220 approve? Use the normal approximation.

\[ P(x < 220) = P(x < 219.5) = P(z < -0.49) = 0.3121 \]

\[ n \cdot p = 500 \cdot 0.45 = 225, \quad \sigma = \sqrt{n \cdot p \cdot (1-p)} = \sqrt{500 \cdot 0.45 \cdot 0.55} = 11.12 \]

\[ z = \frac{x - \mu}{\sigma} = \frac{219.5 - 225}{11.12} = -0.49 \to z \to -0.49 \to \text{Table A1} \to 0.3121 \]

2. You survey 500 people and find that 271 approve of the president. Give a 97% confidence interval for the approval rating. (Approval rating = population proportion)

\[ E = \frac{z_{\alpha/2} \cdot \sqrt{\hat{p} \cdot (1-\hat{p})}}{\sqrt{n}} = 2.17 \cdot \sqrt{\frac{0.542 \cdot 0.458}{500}} = 0.048 \]

\[ \alpha = 1 - \text{Conf. level} = 1 - 0.97 = 0.03 \quad \hat{p} = \frac{x}{n} = \frac{271}{500} = 0.542 \]

\[ 1 - \frac{z_{\alpha/2}^2}{2} = 1 - 0.015 = 0.9850 \quad \hat{p} = 1 - \hat{p} = 0.458 \]

\[ \text{Table A1} \]

\[ z_{\alpha/2} = 2.17 \]

\[ \{ \text{Conf. Int.} \quad \hat{p} - E < \hat{p} < \hat{p} + E \} \]

\[ 0.442 - 0.048 < \hat{p} < 0.542 + 0.048 \]

\[ 0.494 < \hat{p} < 0.590 \]

We are 97% confident that the approval rating is between 49.4% and 59.0%.

3. What sample size would bring this to a 3% error with 95% confidence? (\( \alpha = 1 - 0.95, \quad 1 - \frac{\alpha}{2} = 0.9750 \))

\[ n = \frac{z_{\alpha/2}^2 \cdot \hat{p} \cdot (1-\hat{p})}{E^2} = \frac{1.96^2 \cdot 0.542 \cdot 0.458}{0.03^2} = 1059.6 \to 1060 \]

\[ z_{\alpha/2} = 1.96 \]